Foundation of Risk Management

Delineating Efficient Portfolios

1. $E(R_P) = \sum W_i R_i = W_A E(R_A) + W_B E(R_B)$
2. $\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B Cov(A, B)$

   Where, $Cov(A, B) = \sigma_A \sigma_B r_{(A,B)}$
3. $\rho_{xy} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$
4. When $\rho = 1$,

   $\sigma_P = W_A \sigma_A + W_B \sigma_B$
5. When $\rho = 0$,

   $\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2$
6. When $\rho = 0$, weights of the minimum variance portfolio:

   $w_A = \frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}$

   $w_B = 1 - w_A$
7. When $\rho = -1$, weights of a 'zero' variance portfolio:

   $w_A = \frac{\sigma_B}{\sigma_A + \sigma_B}$

   $w_B = 1 - w_A$
8. *Capital Allocation Line:*

   $E(R_P) = W_{RF} R_F + W_A E(R_A)$

   $\sigma_P = W_A \sigma_A$

   $E(R_P) = R_F + \frac{R_A - R_F}{\sigma_A} \times \sigma_P$
9. *Capital Market Line:*

   $E(R_P) = W_{RF} R_F + W_M E(R_M)$

   $\sigma_P = W_M \sigma_M$

   $E(R_P) = R_F + \frac{R_M - R_F}{\sigma_M} \times \sigma_P$

The Standard Capital Assets Pricing Model

1. $E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right] \times \sigma_P$, as per CAL.
2. $\beta_i = \frac{Cov_{(i,m)}}{\sigma_m^2} = \rho_{i.m} \times \frac{\sigma_i}{\sigma_m}$
3. $\rho_{i,m} = \frac{Cov_{(i,m)}}{\sigma_i \sigma_m}$
4. *CAPM:* $R_e = R_F + (R_M - R_F) \beta$, *as per CAPM.*
5. *For 'n' equally weighted asset:*

   $\sigma_P^2 = \frac{\overline{\sigma^2}}{n} + \frac{(n-1)}{n} \overline{cov} = \frac{var - cov}{n} + Cov$, *for unequally weighted assets.*

Applying the CAPM to Performance Measurement: Single -Index Performance Measurement Indication

1. Treynor Ratio $= \left[\frac{E(R_P) - R_F}{\beta_P}\right]$
2. Sharpe Measure = $\left[\frac{E(R_P) - R_F}{\sigma_P}\right]$
3. Jensen's $\alpha = E(R_P) - [R_F + [E(R_M) - R_F]\,\beta_P]$

   Extension to Jensen's α:

   $E(R) = R_F + [E(R_M) - R_F][\frac{\sigma_P}{\sigma_M}]$

   The alpha in the case would be the portfolio's return minus the reference return:

   $\alpha = E(R_P) - E(R_{reference})$

4. Information Ratio $= \left[\frac{E(R_P - R_B)}{tracking\ error}\right] = \frac{active\ return}{\text{active risk}}$, Tracking error $= \frac{\sqrt{\Sigma(R_P - R_B)^2}}{n-1}$
5. Sortino Ratio $= \frac{R_P - R_{min}}{downside\ deviation}$

   Where, $MSD_{min} = \frac{\Sigma(R_{pt} - R_{min})^2}{N}$

6. Sharpe $\approx \left[\frac{\text{Treynor measure}}{\sigma_M}\right]$ for well diversified portfolio.

Arbitrage Pricing Theory and Multifactor Models

1. Multifactor Model:

$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$

2. Single Factor Security Market Line:

$E(R_P) = [R_F + \beta_P[E(R_M) - R_F]$

3. $E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \ldots \beta_{ik}RP_k$, as per the Arbitrage Pricing Theory.
4. Fama-French Three-Factor Model:

$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB} \times SMB + \beta_{i,HML} \times HML + e_i$

Quantitative Analysis

Time Value of Money

1. Single Cash Flow:

   $FV = PV\left(1+\frac{r}{m}\right)^{n\times m}$

   $PV = \frac{FV}{(1+{}^{r}/_{m})^{n\times m}}$

2. Perpetuity:

   PV = $\frac{PMT}{{}^{I}/_{Y}}$

3. Uneven Cash Flow:

   PV = $\frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n}$

4. Outstanding Loan at any point of time = PV of remaining PMTs

Probabilities

1. Probability = $\frac{\text{No. of favourable outcome}}{\text{Total no. of possible outcome}}$

2. ∑P = 1 (For all exhaustive events)

   $0 \leq P \leq 1$

3. P(A∩B) = P(A|B) × P(B)

4. P(A|B) = $\frac{P(AB)}{P(B)}$

5. Addition Rule:

   P(A∪B) = P(A)+ P(B) - P(A∩B)

6. Multiplication Rule:

   P(A∩B) = P(A)× P(B)

7. For mutually exclusive events:

   P(A∪B) = P(A)+ P(B)

   ∵P(A∩B) = 0

Basic Statistics

1. Population:

   $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$

2. Sample: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

3. Sum of mean deviation:

   $\sum_{i=1}^{n}(X_i - \bar{X}) = 0$

4. $GM = [(1+r_1)(1+r_2)\ldots\ldots(1+r_n)]^{1/n} - 1$

5. AM≥GM≥HM

6. $E(X) = \Sigma\, P(X_1)X_1 + P(X_2)X_2 + \ldots + P(X_n)X_n$ = ∑P(X).X

7. $\rho_{A,B} = \frac{cov(A,B)}{\sigma_A \sigma_B}$

8. If c is any constant, then: E(cX) = cE(X)

9. If X and Y are any random variables, then: E(X+Y) = E(X)+E(Y)

Basic Statistics

10. If c and a are constant then:
    $E(cX + a) = cE(X) + a$
11. If X and Y are independent random variables, then:
    $E(XY) = E(X) \times E(Y)$
12. If X and Y are not independent, then:
    $E(XY) \neq E(X) \times E(Y)$
13. If X is a random variable, then:
    $E(X^2) \neq [E(X)]^2$
14. $\sigma^2 = E[(R - \mu)^2]$
15. Properties of variance include:
    $Var(X) = E[(X - \mu)^2] = E(X^2) - [E(X)^2]$
16. If c is any constant, then:
    $Var(c) = 0$
17. If c is any constant, then:
    $Var(cX) = c^2 \times Var(X)$
18. If c is any constant, then:
    $Var(X + c) = var(X)$
19. If a and c are any constant, then:
    $Var(aX + c) = a^2 \times Var(X)$
20. If X and Y are independent random variables, then:
    $Var(X + Y) = Var(X) + Var(Y)$
    $Var(X - Y) = Var(X) + Var(Y)$
21. If X and Y are independent and a and c are constant, then:
    $Var(Ax + cY) = a^2 \times Var(X) + c^2 \times Var(Y)$
22. $Cov(R_i, Rj) = E\{[Ri - E(Ri)] - \{[Rj - E(Rj)]\}$
23. $Cov(Ri, Rj) = E(Ri, Rj) - E(Ri) \times E(Rj)$
24. If X and Y are independent random variables, then:
    $Cov(X, Y) = 0$
25. The covariance of random variable X with itself is the variance of X.
    $Cov(X, X) = \sigma_x^2$
26. If a, b, c, d are constant, then:
    $Cov(a + bX, c + dY) = b \times d \times Cov(X, Y)$
27. If X and Y are not independent, then:
    $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$
    $Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y)$
28. $E(R) = \mu = \Sigma PX$
29. $\text{Skewness} = \frac{E[(R-\mu)^3]}{\sigma^3}$
30. $\text{Kurtosis} = \frac{E[(R-\mu)^4]}{\sigma^4}$
31. Excess Kurtosis = Kurtosis – 3

Distributions

1. The Binomial Distribution:
   $^{n}C_{x}\, p^{x}\,(1-p)^{n-x}$
   Where, $^{n}C_{x} = \frac{n!}{(n-x)!x!}$
2. E(X) = np
3. $\sigma_x^2 = npq = np(1-p)$
4. Poisson Distribution: $P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$
5. Normal Distribution:
   Z = $\frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x-\mu}{\sigma}$
6. Chi - squared Distribution: $X^2 = \frac{(n-1)S_x^2}{\sigma^2}$
7. F - Distribution: $F = \frac{S_1^2}{S_2^2}$, $S_1 > S_2$ always
8. Central limit Theorem:
   If $n \geq 30$,
   $E(\overline{X}) = \mu$
   For Standard Error: $\sigma_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}}$ or $\frac{S_x}{\sqrt{n}}$ if 'σ' not known.
9. Uniform distribution range: $P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$
10. PDF of continuous uniform distribution: f(x) = $\frac{1}{b-a}$ for $a \leq x \leq b$, else f(x) = 0
11. Mean of uniform distribution: E(x) = $\frac{a+b}{2}$
12. Variance of uniform distribution: Var(x) = $\frac{(b-a)^2}{12}$
13. Binomial probability function: p(x) = $\frac{n!}{(n-x)!x!}\, p^x$
14. Expected value of binomial random variable: expected value of X = E(X) = np
15. Variance of binomial random variable: variance of X = np(1 – p)

Bayesian Analysis

1. $P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$

Characterizing Cycles

1. $Ly_t = y_{t-1}$
2. $\Delta y_t = (1-L)y_t = y_t - y_{t-1}$
3. $\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$
4. $\hat{\rho}(T) = \frac{\sum_{t=\tau+1}^{T}[(y_t-\bar{y})(t_{t-\tau}-\bar{y})]}{\sum_{t=1}^{T}((y_t-\bar{y})^2}$

Volatility

1. EWMA:
   $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)\mu_{n-1}^2$
2. GARCH (1,1)
   $\sigma_n^2 = \omega + \alpha\mu_{n-1}^2 + \beta\sigma_{n-1}^2$; $\omega = \gamma V_L$
   Where, VL = long-run Variance $= \frac{\omega}{1-\alpha-\beta}$
   $\alpha + \beta + \gamma = 1$

Correlations and Copulas

1. $r_{x_1y} = \frac{Cov(x_1 y)}{\sigma_x \sigma_y}$
2. EWMA :
   $Cov_n = \lambda Cov_{n-1} + (1-\lambda)X_{n-1}y_{n-1}$
3. GARCH (1,1):
   $Cov_n = \omega + \alpha X_{n-1}y_{n-1} + \beta Cov_{n-1}$
   Where $\omega = \gamma \times$ Long term covariance.

Hypothesis Testing and Confidence Intervals

1. $Z_{\alpha/2}$ = 1.65 (90% Confidence Interval)
   $Z_{\alpha/2}$ = 1.96 (95% Confidence Interval)
   $Z_{\alpha/2}$ = 2.58 (99% Confidence Interval)
2. Confidence Interval = Point Estimate ± (Reliability Factor × Standard Error)
   Or, Confidence Interval = $\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
3. H_a: Alt. Hypothesis;
   H_0: Null Hypothesis
4. Test Statistic = $\frac{\text{Sample statistic} - \text{hypothesized value}}{\text{Standard error of the sample statistics}}$
5. t-Test:
   $t_{n-1} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
6. z-Test:
   $z_{test} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
7. Chi-square test:
   H_0: $\sigma^2 = \sigma_0^2$ Vs H_A: $\sigma^2 \neq \sigma_0^2$
8. $\mathcal{X}^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$
9. F- test:
   $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_a : \sigma_1^2 \neq \sigma_2^2$
10. $F_{Stat} = \frac{S_1^2}{S_2^2}$
11. Type I error: The rejection of the null hypothesis when it is actually true. [HORN]
12. Type II error: The failure to reject the null hypothesis when it is actually false.
13. P(Type I error) = α
14. P(Type II error) = 1 - Power of the Test

Linear Regression with One Regressor

1. Y = b_0+b_1X; where y is dependent variable. X is independent variable and b_0 and b_1 is regression coefficient.
2. $E(Y_i|X_i) = B_0 + B_1 \times X_i$
3. $\varepsilon_i = Y_i - E(Y_i|X_i)$ Or,
   $\varepsilon_i = (Y_i - \hat{Y})$
4. $Y_i = B_0 + B_1 \times X_i + \varepsilon_i$
5. OLS: Minimize $\sum e_i^2 = \sum[Y_i - (b_0 + b_1X_i)]^2$
   or, $\sum(Y_i - \hat{Y})^2 = \sum[Y_i - (b_0 + b_1X_i)]^2$
6. $b1 = \frac{Cov(x,y)}{Var(x)}$ and $bo = \bar{Y} - b_1\bar{X}$
7. Total sum of squares [TSS] = Explained sum of squares [ESS] + Sum of squared Residuals[SSE]
   Or, TSS = RSS + SSR
   Or, $\sum(Y_i - \bar{Y})^2 = \sum(\hat{Y} - \bar{Y})^2 + \sum(Y_i - \hat{Y})^2$
8. $R^2 = \frac{ESS}{TSS} = \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2}$
   Or, $\frac{TSS - SSR}{TSS}$ or 1 - $\frac{SSR}{TSS}$

Regression with a Single Regressor: Hypothesis Tests and Confidence Intervals

1. Confidence interval for the regression coefficient, B_1:
   $b_1 \pm (t_c \times \sigma_{b_i})$
2. Test statistics with n-2 degrees of freedom is:
   $t = \frac{b_1 - B_1}{s_{b_i}}$
3. $H_0 : B_1 = 0$ versus $H_A : B_1 \neq 0$
4. The predicted value of Y is:
   $Y = b_0 + b_1 X$
5. Confidence interval for a predicted value of Y is:
   $\hat{Y} - (t_c \times \sigma_f) < Y < \hat{Y} + (t_c \times \sigma_f)$

Hypothesis Testing and Confidence Intervals in Multiple Regression

1. $\text{t statistic} = \frac{\text{Estimated regression} - \text{hypothesized value}}{\text{coefficient st. error}}$

The statistic has $n - k - 1$ degrees of freedom

2. Testing Statistical Significance:

$H_0 : b_j = 0$ versus $H_A : b_j \neq 0$

3. Confidence interval for the regression coefficient:

$b_j \pm (t_c \times s_{b_j})$

4. Predicting the dependent variable:

$\hat{Y}_i = b_0 + b_1\hat{X}_{1i} + b_2\hat{X}_{2i} + \cdots + b_k\hat{X}_{ki}$

5. For Joint Hypothesis using F_{stat}:

$H_0 : B_1 = B_2 = B_3 = B_4 = 0$ vs H_A : at least one $b_j \neq 0$

6. Homoskedasticity only $f_{stat} = \frac{ESS/df_n}{SSR/df_d}$

$df_{numerator} = k$

$df_{denominator} = n - k - 1$

7. Coefficient of Determination:

$R^2 = \frac{ESS}{TSS} = \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2}$

8. Adjusted $R^2 = 1 - \left[\left(\frac{n-1}{n-k-1}\right) \times (1 - R^2)\right]$

Modelling and Forecasting Seasonality

1. A pure seasonal dummy model:

$$y_t = \sum_{i=1}^{s} \gamma_i (D_{i,t}) + \varepsilon_t$$

2. Adding a trend:

$$y_t = \beta_1(t) + \sum_{i=1}^{s} \gamma_i (D_{i,t}) + \varepsilon_t$$

3. Allowing for holiday variations (HDV) and trading day variations (TDV):

$$y_t = \beta_1(t) + \sum_{i=1}^{s} \gamma_i (D_{i,t}) + \sum_{i=1}^{v_1} \delta_i^{HDV}(HDV_{i,t}) + \sum_{i=1}^{V_2} \delta_i^{TDV}(TDV_{i,t}) + \varepsilon_t$$

$$y_{T-h} = \beta_1(T + h) + \sum_{i=1}^{s} \gamma_i (D_{i,T+h}) + \sum_{i=1}^{v_1} \delta_i^{HDV}(HDV_{i,T+h}) + \sum_{i=1}^{V_2} \delta_i^{TDV}(TDV_{i,T+h}) + \varepsilon_t$$

Financial Markets & Products

Insurance Company & Pension Plans

1. Combined Ratio = Loss ratio + Expense Ratio
2. C.R after dividends = Combined Ratio + Dividends
3. Operating Ratio = Combined Ratio after dividends – Investment Income

Mutual Funds & Hedge Funds

1. NAV = $\frac{\text{fund assets} - \text{fund liabilities}}{\text{total share outstanding}}$

Introduction – Option, Futures and Other Derivatives

1. Call option payoff:
   $C_T = Max(0, S_T - x)$
   Profit to option buyer = $C_T - C_0$
   Profit to option seller = $C_0 - C_T$
2. Put option payoff:
   $P_T = Max(0, X - S_T)$
   Profit to option buyer $= P_T - P_0$
   Profit to option seller = $P_0 - P_T$
3. Forward contract payoff:
   Payoff to a long position $= S_T - k$
   Pay off to a short position = $k - S_T$

Hedging Strategies using Futures

1. Basis = Spot Price – Future price = $S_t - F_0$
2. H.R = $\rho_{s,f} \times \frac{\sigma_s}{\sigma_f}$
   Effectiveness of Hedge: $R^2 = \rho^2$
3. Correlation: $\rho = \frac{cov_{s,f}}{\sigma_s \sigma_f}$
   and $\frac{cov_{s,f}}{\sigma_s \sigma_f} \times \frac{\sigma_s}{\sigma_f} = \frac{cov_{s,f}}{\sigma_{f2}} = \beta_{S,F}$
4. Hedging with stock index futures:
   $N = \beta_P \times \left(\frac{\text{Portfolio Value}}{\text{Value of the future contract}}\right)$
   $= \frac{V_P(\beta_T - \beta_P)}{m \times F_P \times \beta_f}$
5. Adjusting portfolio beta: number of contracts = $(\beta^* - \beta)\frac{P}{A}$
6. For hedging the tail:
   $\Rightarrow n \times \frac{\text{daily spot}}{\text{daily futures}}$

Interest Rates

1. Discrete: FV = $A\left(1+\frac{R}{m}\right)^{m\times n}$
2. Continuous: FV = $Ae^{R\times n}$
3. Bond pricing:

$$B = \left(\frac{c}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2}\times j}\right)$$

where:

c = the annual coupon

N = the number of semiannual payment periods

z_j = the bond equivalent spot rate that corresponds to j periods (j/2 years) on a continuously compounded basis

FV = the face value of the bond

4. Using above two equations:

$$A\left(1+\frac{R}{m}\right)^{m\times n} = Ae^{R_c n}$$

$$\therefore R_C = m \times l_n\left(1+\frac{R}{m}\right)$$

5. Forward Rate Agreements:

Cash flow (if receiving R_K) = L × (R_K – R) × (T_2 – T_1)

Cash flow (if paying R_K) = L × (R – R_K) × (T_2 – T_1)

where:

L = principal

R_K = annualized rate on L, expressed with compounding period T_2 – T_1

R = annualized actual rate, expressed with compounding period T_2 – T_1

T_i = time i, expressed in years

Payoff = $\frac{(\text{Mkt.rate} - \text{contract rate}) \times \frac{n}{12} \times \text{NP}}{1+(\text{Mkt.rate} \times \frac{n}{12})}$

Percentage bond price change ≈ duration effect + convexity effect

Determination of Forward and Futures

1. Forward Price = S x $(1+ r)^t$
2. Forward Price: $F = S_0e^{rt}$
3. With benefits: $F = (S_0 - I)e^{rt}$
4. With dividend: F = S × $[(1 + r) / (1 + q)]^T$

   Currency Futures: $F_0 = S_0e^{(r_{DC} - r_{FC})T}$
5. With income: F = (S – I) × $(1 + r)^T$

Interest Rate Futures

1. A.I = coupon $\times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$
2. Cash Price = Quoted Price + Accrued Interest
3. Quoted Price = Cash Price - Accrued Interest
4. Clean Price = Dirty Price - Accrued Interest
5. Annual rate on a T-bill: T-bill discount rate = $\frac{360}{n}(100 - Y)$
6. BDY = $\frac{\text{FV} - \text{Cash Price}}{\text{FV}} \times \frac{360}{n}$
7. C.T.D = QBP – (QFP × CF)
8. Conversion Factor = $\frac{\text{Bond Price} - \text{Accrued Interest}}{\text{Face Value}}$
9. Cash received by the short = (QFP × CF) +AI
10. Euro dollar future prices = $\$10{,}000[100 - (0.25)(100 - Z)]$
11. Duration-based H.R

    $N = \frac{V_P(D_T - D_P)}{F \times D_F}$

Swaps

1. $R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$
2. Interest rate swap value: $V_{swap} = Bond_{fixed} - Bond_{floating}$
3. Currency swap value: $V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$

Properties of Stock Options

1. Put - Call Parity: $c + Xe^{-rt} = S + p$

   $S = c - p + Xe^{-rt}$

   $P = c - S + Xe^{-rt}$

   $C = S + p + Xe^{-rt}$

   $Xe^{-rt} = S + p - c$
2. Relationship between American Call and Put Options:

   $S_0 - X \leq C - P \leq S_0 - Xe^{-rt}$
3. Widest possible range after considering dividend:

   $S_0 - X - D \leq C - P \leq S_0 - Xe^{-rt}$

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}

Exotic Options

1. Cash-or-nothing = $\frac{QN(d_2)}{e^{rt}}$
2. Asset-or-nothing = $\frac{S_0 N(d_2)}{e^{qt}}$

Commodity Forward & Futures

1. Commodity Forward Price: $F_{0,T} = E(S_T)e^{(r-\alpha)T}$
2. NPV = $E(S_T)e^{-\alpha T} - S_0$
3. Commodity Forward Price with active lease market: $F_{0,T} = S_0 e^{(r-\delta)T}$ or $S_0 \times [(1 + r) / (1 + \delta)]^T$
4. With Storage Cost: $F_{0,T} = S_0 e^{(r+\lambda)T}$ *or* $(S_0 + U) \times (1 + r)^T$
5. With Convenience Yield: $F_{0,T} = S_0 e^{(r-c)T}$ *or* $F_{0,T} \geq (S_0 + U) \times [(1 + r) / (1 + Y)]^T$
6. Combination of cost & benefits:

   $F_{0,T} = S_0 e^{(r+\lambda-c)T}$
7. Arbitrage free range of the forward price:

   $S_0 e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0 e^{(r+\lambda)T}$

Foreign Exchange Risk

1. Net EUR exposure = (EUR assets – EUR liabilities) + (EUR bought – EUR sold)
Net EUR exposure = Net EUR assets + net EUR bought.

2. Dollar gain / loss in EUR = Net EUR exposure (measured in \$) × % change in \$/FC rate.

3. Purchasing power parity:

$\%\Delta S$ = inflation(foreign) – inflation(domestic)

where: $\%\Delta S$ = change in the domestic spot rate

4. IRP: $\text{Forward} = \text{Spot}\left[\frac{(1+r_{YYY})}{(1+r_{XXX})}\right]^{T}$

Or,

$\text{Forward} = \text{Spot} \times e^{(r_{YYY}-r_{xxx})T}$

where: r_{YYY} = quote currency
rate r_{XXX} = base currency rate

5. The nominal interest rate:
Exact methodology: (1+Nominal Rate) = (1+ Real Rate) (1+ Inflation Rate)
Linear approximation: Nominal Rate ≈ Real Rate + Inflation Rate

Corporate Bonds

1. Original-issue discount (OID) = face value – offering price

2. Issue default rate = $\frac{\text{No.of issuers defaulted}}{\text{Total no.of issuers at the beginning of the year}}$

3. Dollar default rate = $\frac{\text{Cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance})\times(\text{weighted average \# of years outstanding})}$

   Or, $\frac{\text{Cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$

4. Expected loss rate = probability of default × (1 – expected recovery rate)

Mortgages & Mortgage-Backed Securities

1. $SMM = 1 - (1 - CPR)^{1/12}$
2. $CPR = 1 - (1 - SMM)^{12}$
3. Option cost = Zero volatility spread – OAS
4. Value of a dollar roll = A – B + C – D
   A = Price at which pool is sold in month 1, with accrued interest
   B = Price at which pool is bought in month 2, with accrued interest
   C = Interest earned on funds from the sale for one month
   D = Coupon and principal payment that was foregone on the pool sold in month 1

Trading Strategies

1. Bull call spread: profit = $\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$
2. Bear put spread: profit = $\max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$
3. Butterfly spread with calls:
   profit = $\max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$
4. Straddle: profit = $\max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$
5. Strangle: profit = $\max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

Valuation & Risk Model

VaR Methods

1. VaR (%) = $\overline{X} - (Z_{stat} \times \sigma)$
2. Mean: $\mu_P = w_1 \mu_1 + w_2 \mu_2$
3. Standard deviation: σP = $\sqrt{w_1^2\,\sigma_1^2 + w_2^2\,\sigma_2^2 + 2_\rho\, w_1\, w_2\, \sigma_1\, \sigma_2\, \rho}$
4. VaR ($) = VaR (%) × V_P
5. Delta-normal VAR: VaR = $[\mu - Z_{stat}.\sigma] \times$ portfolio value
6. Expected shortfall: ES =$(\mu + \sigma \frac{e^{-(z^2/2)}}{(1-x)\sqrt{2\pi}}$
7. VaR = modified duration x Z x annualized yield volatility x portfolio value
8. delta: δ =$\frac{\Delta P}{\Delta S}$
9. VaR(T,X) = VaR(1,X) × √T
10. ES(T,X) = ES(1,X) × √ _ T
11. $\sigma_{daily} \cong \frac{\sigma_{annual}}{\sqrt{250}}$ or,

    $\sigma_{monthly} \cong \frac{\sigma_{annual}}{\sqrt{12}}$

Quantifying Volatility in VaR Models

1. Parametric Approach: $r^2_{t-k,t-k-1} = (r^2_{t-3,t-2} + \cdots + r^2_{t-2,t-1} + r^2_{t-1,t})$
2. Exponentially weighted moving average (EWMA) model: $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)r_{n-1}^2$
3.
4. (1,1):

   $\sigma_t^2 = \omega + \alpha r^2_{t-1,t} + \gamma\sigma^2_{t-1}$
5. Risk Metrics(R) Approach:

   $\sigma_t^2 = (1-\lambda)(\lambda^0 r^2_{t-1,t} + \lambda^1 r^2_{t-2,t-1} + \lambda^2 r^2_{t-3,t-2} + \cdots + \lambda^N r^2_{t-N-1,t-N})$
6. Hybrid Approach:

   Weightage to 't' return:

   $r_t = \left[\frac{1-\lambda}{1-\lambda^k}\right]\lambda^{t-1}$
7. MDE: $\sigma_t^2 = \sum_{i=1}^{K} \varpi\,(x_{t-1} - i)r^2_{t-i}$

Putting VaR to Work

1. Taylor Series approximation: $f(x) \approx f(x_0) + f^1(x - x_0) + \frac{1}{2} f''(x_0)\,(x - x_0)^2$

Binomial Trees

1. HR = $\frac{C_U - C_D}{S_U - S_D}$
2. Call price = hedge ratio x [stock price – PV (borrowing)]
3. U = size of the up − move factor = $e^{\sigma\sqrt{t}}$

   D = size of the down − move factor = $e^{-\sigma\sqrt{t}} = \frac{1}{e^{\sigma\sqrt{t}}} = \frac{1}{U}$
4. $\pi_U = \frac{e^{rt} - D}{U - D}$

   $\pi_d = 1 - \pi_U$
5. $\pi_u = \frac{e^{(r-q)t} - D}{U - D}$, In case a stock pays dividend

   $\pi_d = 1 - \pi_u$
6. $\pi_u = \frac{e^{(r_{DC} - r_{FC})t} - D}{U - D}$, In case of currencies
7. $\pi_u = \frac{1 - D}{U - D}$, In case of options on futures

Prices, Discounted Factors & Arbitrage

1. A.I = Coupon payment $\times \left(\frac{\text{No.of days from last coupon settlement}}{\text{No.of days in coupon period}}\right)$
2. P = $\frac{C}{(1+Y)^{w}} + \frac{C}{(1+Y)^{1+w}} + \frac{C}{(1+Y)^{2+w}} + \frac{C}{(1+Y)^{n-1+w}} + \frac{M}{(1+Y)^{n-1+w}}$
3. Flat Price = Full Price – A.I
4. Clean price = dirty price – accrued interest

The Black Scholes Merton Model

1. $E(S_T) = S_0 e^{\mu T}$
2. Valuation of warrants: $\frac{N}{N+M}$ × value of regular call option
3. Continuously compounded returns: $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$
4. BSM Option Pricing Model:

   $C_0 = [S \times N(d_1)] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$

   $P_0 = \left\{X \times e^{-R_f^c \times T} \times [1 - N(d_2)]\right\} - \{S_0 \times [1 - N(d_1)]\}$

   Where,

   $d_1 = \frac{\left[\ln\left(\frac{S_0}{X}\right) + \left\{R_f^c + (0.5 \times \sigma^2)\right\}\right] T}{\sigma\sqrt{T}}$

   $d_1 = d_1 - (\sigma \times \sqrt{T})$

The Greek Letters

1. Delta: $\Delta = \frac{\delta c}{\delta s}$
2. No. of options needed to delta hedge = $\frac{\text{No.of shares hedged}}{\text{delta of call option}}$
3. ΔValue of puts = Δ Value of long stock option
4. Portfolio delta: $\Delta_p = \sum_{i-1}^{n} w_i \Delta_i$
5. gamma = $\frac{\partial^2 c}{\partial s^2}$
6. vega = $\frac{\partial c}{\partial \sigma}$ = S0N' (d1)
7. rho = $\frac{\partial c}{\partial r}$
8. theta: $\theta = \frac{\partial c}{\partial t}$
9. Relationship among delta, theta, and gamma: $\prod r = \theta + rS\Delta + 0.5\sigma^2 S^2 \gamma$

Returns, Spreads & Yields

1. HPR: $R_{t-1,t} = \frac{BV_t + C_t - BV_{t-1}}{BV_{t-1}}$
2. PV of Perpetuity = $\frac{C}{y}$
3. Bond Price:

   $P = \frac{c_1}{(1+y)^1} + \frac{c_2}{(1+y)^2} + \frac{c_3}{(1+y)^3} + \ldots + \frac{c_n}{(1+y)^n}$

Spot, Forwards & Par Rates

1. $FV_n = PV_0 \times \left[1 + \frac{r}{m}\right]^{m \times n}$
2. Compounding frequencies: $R_2 = \left[\left(1 + \frac{R_1}{m_1}\right)^{m_1/m_2} - 1\right] m_2$
3. Holding Period Return = $\frac{P_n - P_0 + C.F}{P_0}$
4. Forward rate: $F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$
5. EAY = $m[(1 + HPY)^{m \times n} - 1]$
6. Spot Rate: $Z_{(t)} = 2\left[\left(\frac{1}{d(t)}\right)^{1/2t} - 1\right]$
7. Discount factor: $d(t) = \left(1 + \frac{r(t)}{2}\right)^{-2t}$
8. Discount Rate: $d(n) = \frac{1}{(1+S_n)^n}$
9. Par Rate: $x = \frac{1-d_2}{\sum d}$

One-Factor Risk Metrics & Hedges

1. DV01 = $\frac{\Delta P}{\Delta y}$
2. HR = $\frac{\text{DV01(per \$100 of initial position)}}{\text{DV01(per \$100 of hedging instrument)}}$
3. Macaulay's Duration = $\frac{\sum wx}{\sum x}$
4. Duration Gap = Macaulay's Duration – Investment Horizon
5. Effective Duration = $\frac{\Delta P}{P\Delta y}$
6. Effective Convexity = $\frac{(P_2 + P_1 - 2P_0)}{P_0(\Delta y)2}$
7. Effect of EC = $\frac{1}{2} \times EC \times (\Delta y)^2$
8. $\text{Modified Duration} = \frac{\text{Macaulay's Duration}}{\text{1+periodic market yield}}$
9. MD $\simeq$ ED [for all option free bonds]
10. DV01 = Duration $\times 0.0001 \times$ Bond Value
11. Percentage Price Change $\approx$ Duration effect + Convexity Effect

    $= [-\text{duration} \times \Delta y \times 100] + \left[\left(\frac{1}{2}\right) \times \text{convexity} \times (\Delta y)^2 \times 100\right]$
12. $D_{portfolio} = \sum_{i=1}^{k} W_i D_i$
13. $\Delta P = -D \times P \times \Delta y + \frac{1}{2} \times C \times P \times \Delta y^2$

Multi-Factor Risk Metrics & Hedges

1. Key Rate 01 = $-\frac{1}{10{,}000} \frac{\Delta BV}{\Delta y^k}$
2. Key Rate duration = $-\frac{1}{BV} \frac{\Delta BV}{\Delta y^k}$

Capital Structure in Banks

1. E.L = PD × LGD × EAD
2. Standard deviation of credit loss: σ = $\sqrt{\mathrm{PD} - \mathrm{PD}^2}$ × [L(1 – RR)]
3. Standard deviation of credit loss as percentage of size: α = $\frac{\sigma_P}{nL} = \frac{\sigma\sqrt{1+(n+1)\rho}}{\sqrt{n}\times L}$
4. Unexpected loss: UL = (WCDR – PD) × LGD × EAD
5. UL = EAD × $\sqrt{(\mathrm{PD} \times \sigma^2_{\mathrm{LGD}}) + (\mathrm{LGD}^2 \times \sigma^2_{\mathrm{PD}})}$
6. σ^2_{PD} = PD × (1 - PD)
7. ELP = $\sum(\mathrm{EAD_i} \times \mathrm{PD_i} \times \mathrm{LGD_i})$
8. ULP = $\sqrt{\mathrm{UL}_1^2 + \mathrm{UL}_2^2 + 2\rho_{1,2}.\mathrm{UL}_1.\mathrm{UL}_2}$
9. For two assets portfolio:

   $\mathrm{RC}_1 = \frac{\mathrm{UL}_1 + (\rho_{1,2} \times \mathrm{UL}_2)}{\mathrm{UL_p}} \times \mathrm{UL}_1$

   $\mathrm{RC}_2 = \frac{\mathrm{UL}_2 + (\rho_{1,2} \times \mathrm{UL}_1)}{\mathrm{UL_p}} \times \mathrm{UL}_2$

   $\mathrm{RC}_1 + \mathrm{RC}_2 = \mathrm{UL_p}$
10. $\text{Economic Capital}_{\mathrm{P}} = \mathrm{UL_p} \times \text{Capital Multiplier}$